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Modeling of Porous Elastic-Viscoplastic Material with Tensile Failure

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Abstract

This work describes simple but comprehensive constitutive equations that model a number of physical phenomena exhibited by dry porous geological materials and metals. Moreover, formulas have been developed for robust numerical integration of the evolution equations at the element level that can be easily implemented into standard computer programs for dynamic response of materials.

Introduction

Developing realistic mathematical constitutive equations to model the dynamic response of a wide range of materials has been a main objective of research in continuum mechanics for the last few decades. With ever increasing computational power of modern computers it has become more practical to implement these nonlinear constitutive equations in real applications. The response of structures to shock loading is a particularly challenging area of research because the constitutive equations must model fully coupled nonlinear thermomechanical effects near the shock source and must model complicated features of the strength of the material far away from the source where the pressures are relatively low.

Inelasticity in metals is due to dislocation motion whereas inelasticity in geological materials is usually due to microfracturing which accompanies porous compaction and dilation. In spite of these physical differences, the mathematical structure of plasticity theory seems to be capable of capturing the main features of the response of both of these materials. In this paper, a set of constitutive equations is briefly described which models a number of physical phenomena exhibited by dry porous geological materials and metals. The equations are valid for large deformations and the elastic response is hyperelastic in the sense that the stress is related to a derivative of the Helmholtz free energy. Details of these equations can be found in Rubin et al (1998).

Constitutive modeling always requires compromises. On the one hand, attempts are made to include details of relevant physical phenomena and on the other hand, attempts are made to develop mathematically simple equations. For example, sometimes the constitutive equations are formulated in terms of yield surfaces for distortional deformation and compaction surfaces for porosity. Such equations are often difficult to satisfy numerically because they become nonlinear algebraic equations which must be solved iteratively. Here, the equations developed in (Rubin, et al, 1996) are reformulated as rate-type evolution equations. It will be seen that by proper choice of the functional forms for these evolution equations it is possible to retain the main physical features of the algebraic equations and at the same time develop robust numerical algorithms which integrate these evolution equations explicitly.

Basic Thermomechanical Equations

The constitutive equations are developed using the thermodynamical procedures proposed by Green and Naghdi (1977,1978). Within this context, the usual laws of conservation of mass and balances of linear momentum, angular momentum and energy are supplemented by a balance of entropy. The constitutive equations are then restricted so that the local forms of the balances of angular momentum and energy are satisfied for all thermomechanical processes. Also, various statements of the second law of thermodynamics place additional restrictions on the constitutive equations.

The notion of modeling elastic deformation through an evolution equation was introduced by Eckart (1948), and has been discussed more recently by Rubin (1996). Here, the elastic response for an elastically isotropic material is modeled by a measure of volumetric deformation J_e and a measure of distortional deformation $\mathbf{B}_e'$, which are defined by the evolution equations

$$\dot{J}_e = J_e \left[\mathbf{D} \bullet \mathbf{I} - A_p \right] \ , \qquad \dot{\mathbf{B}}_e' = \mathbf{L}\,\mathbf{B}_e' + \mathbf{B}_e'\,\mathbf{L}^T - \tfrac{2}{3}(\mathbf{D} \bullet \mathbf{I})\ \mathbf{B}_e' - \mathbf{A}_p \ , \tag{1}$$

where A_p characterizes the rate of inelastic deformation due to porous compaction and dilation and $\mathbf{A}_p$ characterizes the rate of inelastic distortional deformation. Also, J_e is related to the total volumetric deformation J by the formulas

$$J_e = \frac{1-\phi}{1-\Phi}\, J \ , \qquad \dot{J} = J \left[\mathbf{D} \bullet \mathbf{I} \right] \ , \tag{2}$$

where ϕ is porosity in the present configuration and Φ is its reference value. Furthermore, the quantities A_p and $\mathbf{A}_p$ are given by

$$A_p = \frac{\dot{\phi}}{1-\phi} \ , \qquad \mathbf{A}_p = \Gamma_p \left[\mathbf{B}_e' - \left\{ \frac{3}{\mathbf{B}_e'^{-1} \bullet \mathbf{I}} \right\} \mathbf{I} \right] \ , \qquad \Gamma_p \geq 0 \ , \tag{3}$$

where Γ_p is a nonnegative function that controls viscoplasticity.

For simplicity, the specific (per unit mass) Helmholtz free energy function ψ is assumed to be a function of the form

$$\psi = \psi(J_e, \alpha_1, \theta) \ , \qquad \alpha_1 = \mathbf{B}_e' \bullet \mathbf{I} \ , \tag{4}$$

where α_1 is an invariant measure of elastic distortional deformation and θ is the absolute temperature. Moreover, for a dry porous material the Helmholtz free energy ψ is assumed to equal that ψ_s of the solid matrix material. Under these assumptions it can be shown that the pressure p and the deviatoric part $\mathbf{T}'$ of the Cauchy stress $\mathbf{T}$ are determined by derivatives of ψ, with

$$\mathbf{T} = -\,p\,\mathbf{I} + \mathbf{T}' \ , \qquad \mathbf{T}' \bullet \mathbf{I} = 0 \ . \tag{5}$$

The functional form for ψ can be specified so that the pressure is consistent with a Mie-Gruneisen type equation which is common in shock physics. Also, the evolution of elastic distortional deformation $(1)_2$ is controlled by viscoplasticity using a model of the type proposed by Swegle and Grady (1985) such that

$$\Gamma_p = \Gamma_{p0} \left[\frac{3G}{\sigma_e}\right] \left[\frac{<\sigma_e - Y>}{Y_0}\right]^2 \ , \qquad \sigma_e^2 = \frac{3}{2} \mathbf{T}' \bullet \mathbf{T}' \ ,$$

$$<x> = x \quad \text{for } x \geq 0 \ , \qquad <x> = 0 \quad \text{for } x < 0 \ , \tag{6}$$

where Γ_{p0} is a material constant, G is the shear modulus, Y is the yield strength in uniaxial stress, Y_0 is a constant, σ_e is the von Mises stress and $< x >$ are McAuley brackets. Specifically, the yield strength Y is assumed to be a multiplicative function of the form

$$Y = Y_0 \, F_1(\varepsilon_p) \, F_2(p) \, F_3(\Omega) \, F_4(\beta,p) \, F_5(J_e,\theta) \, F_6(\phi) \, F_7(\omega,p) \ . \tag{7}$$

The function F_1 depends on an equivalent plastic strain ε_p and controls hardening; F_2 controls the dependence on pressure; F_3 controls the dependence on a measure Ω of damage due to distortional inelasticity; F_4 controls the dependence on the Lode angle which characterizes the state of deviatoric stress; F_6 controls the dependence on porosity; and F_7 controls the dependence on a measure ω of damage due to porosity changes during dilation or compaction.

Evolution equations for compaction and dilation of porosity have been proposed in (Rubin, et al, 1998) for all modes of deformation. For example, one of these evolution equations for compaction is given by

$$\dot{\phi} = -\Gamma_c < \phi - \phi_c^* > \leq 0 \qquad \text{for } \phi_{max} \geq \phi^* \text{ and } p > p_c \ , \tag{8}$$

where Γ_c and ϕ^* are constants, ϕ_{max} is the maximum value of ϕ attained during the loading, p_c is a function characterizing the pressure at which compaction initiates, and ϕ_c^* is a function that characterizes the value of porosity during compaction. The evolution equation (8) has been proposed instead of a compaction surface since the latter is usually a highly nonlinear algebraic expression which must be solved iteratively. In contrast, by assuming the value of ϕ_c^* is constant during a time step Δt, equation (8) can be integrated to give

$$\phi_2 = \phi_c^* + (\phi_1 - \phi_c^*) \exp\{-\Delta t \, \Gamma_c\} \ , \tag{9}$$

where ϕ_1 is the value of ϕ at the beginning of the time step and ϕ_2 is its value at the end of the time step. This represents a robust solution of the evolution equation (8) because it always ensures that porosity ϕ approaches the value ϕ_c^*. In this regard, it follows that the functional form for ϕ_c^* can be specified to approximate the solution of any compaction surface. Consequently, if the value of Γ_c is large enough then the solution (9) will remain close to the solution of the compaction surface even though (9) is an explicit solution that requires no iteration.

The equations developed in (Rubin, et al, 1998) have been used to simulate experimental data for compaction of Mt. Helen Tuff (Heard, et al., 1973) which is a geological material. They also have been used to match an empirical formula for spall stress which fits experimental data (Kanel et al, 1997) for Aluminum subjected to high uniaxial strain rate. For example, Figure 1a shows a comparison of the model with experimental data of Mt. Helen Tuff for pure dilatational compression. In this figure, E_v is a measure of volumetric strain defined by

$$E_v = J - 1 \ , \tag{10}$$

and c_1 is a material constant that partially characterizes the function ϕ_c^* associated with compaction. Also, Fig. 1b compares the predicted spall stress of Aluminum with the empircal formula (Kanel et al, 1997) for uniaxial strain expansion with deformation rate D_u and axial stress T_{11}. In both of these figures, n is a material constant that controls the spall process.

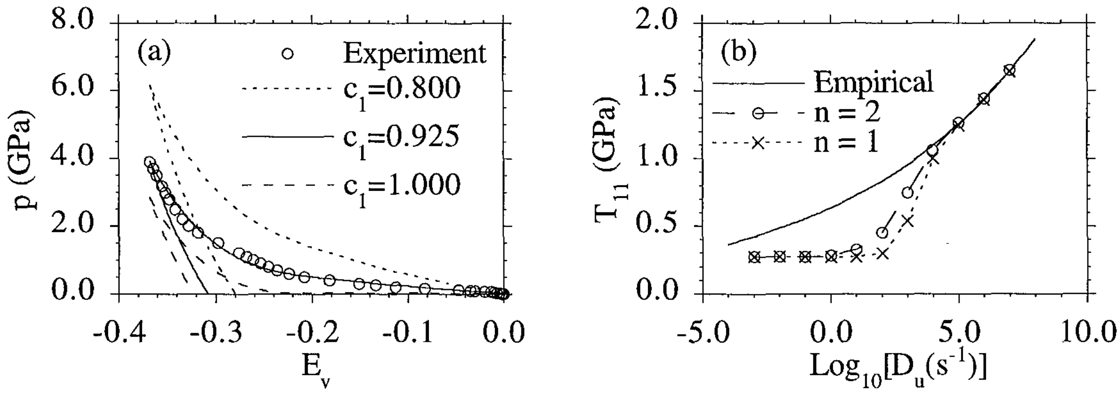

Fig. 1 (a) Pure dilatational compression of a geological material; (b) Spall Stress of Aluminum.

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